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An analytical approximation to the solution of a wave equation by a variational iteration method

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Abstract

In this work, a variational iteration method, which is a well-known method for solving functional equations, has been employed to solve the general form of a wave equation which governs numerous scientific and engineering experimentations. Some special cases of wave equations are solved as examples to illustrate the capability and reliability of the method. The results reveal that the method is very effective. The restrictions of the method are mentioned.

Keywords: Variational iteration method; Wave equation; Correction functional; Restricted variation; Lagrange multiplier

1. Introduction

Wave equations have attracted a great deal of attention and solving different forms of these equations has been an interesting task for mathematicians. Numerical methods which are commonly used such as finite difference and characteristics approaches need large amounts of computational work and usually the effect of rounding-off error causes loss of accuracy in the results. Analytical methods commonly used for solving wave equations are very restricted and can be used only in very special cases, so they cannot be used to solve equations for numerous realistic scenarios. The variational iteration method introduced by He [1–10] has been used by many mathematicians and engineers to solve various functional equations. It has been shown that this procedure is a powerful tool for solving various kinds of functional equations. For example, this scheme is used for solving delay differential equations [10]. Application of the variational iteration method to the Helmholtz equation is investigated in [11]. This technique is used for solving the Burgers and coupled Burgers equations [12]. Applications of the present method to coupled Schrödinger–KdV equations and the shallow water equation are provided [14]. The variational iteration technique is employed for solving an inverse problem [13]. The authors of this work also used this method for solving systems of ordinary differential equations, fourth-order parabolic equations and hyperbolic differential equations [15–17]. Also,

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this method was used to solve other nonlinear equations [18–22]. For the purpose of illustration of the methodology of the variational iteration method, let us consider the following nonlinear functional equation:

$$Lu(t) + Nu(t) = g(t)$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a known analytical function. According to the variational iteration method, we can construct the following correction functional:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\xi)(Lu_n(\xi) + N u_n(\xi) - g(\xi))\mathrm{d}\xi,$$

where λ is a general Lagrange multiplier which can be identified via variational theory, $u_0(t)$ is an initial approximation, with possible unknowns, and $\tilde{u}_n$ is considered as a restricted variation [5,6], i.e. $\delta\tilde{u}_n = 0$. Therefore, we first determine the Lagrange multiplier λ that will be identified optimally via integration by parts. The successive approximations, $u_{n+1}(t)$, to the solution $u(t)$ will be readily obtained upon using the Lagrange multiplier obtained and by using any selected function u_0. Consequently, the exact solution may be obtained by using $u = \lim_{n\to\infty} u_n$.

2. Solution of a wave equation by the variational iteration method

Consider the following general form of a wave equation:

$$\frac{\partial^2 u}{\partial t^2} = A(x, y, z, t)\frac{\partial^2 u}{\partial x^2} + B(x, y, z, t)\frac{\partial^2 u}{\partial y^2} + C(x, y, z, t)\frac{\partial^2 u}{\partial z^2} + D(x, y, z, t). \tag{1}$$

To solve this equation by the variational iteration method, we need to specify the initial or boundary conditions, which for example may be specified as follows:

$$\text{Initial conditions:}\quad u(x, y, z, 0) = f(x, y, z), \quad \frac{\partial u}{\partial t}(x, y, z, 0) = g(x, y, z).$$

$$\text{Boundary conditions:}\quad u(0, y, z, t) = f_1(x, y, z), \quad \frac{\partial u}{\partial x}(0, y, z, t) = g_1(y, z, t),$$

$$\frac{\partial u}{\partial x}(0, y, z, t) = g_2(y, z, t).$$

For solving Eq. (1) by the iterative variation method, its correction functional can be written down as

$$\text{Case 1:}\quad u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \int_0^t \lambda_1 \left(\frac{\partial^2 u_n}{\partial \xi^2} - A\frac{\partial^2 \tilde{u}_n}{\partial x^2} - B\frac{\partial^2 \tilde{u}_n}{\partial y^2} - c\frac{\partial^2 \tilde{u}_n}{\partial z^2} - D\right)\mathrm{d}\xi$$

$$\text{Case 2:}\quad u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \int_0^x \lambda_2 \left(\frac{\partial^2 \tilde{u}_n}{\partial t^2} - A\frac{\partial^2 u_n}{\partial \xi^2} - B\frac{\partial^2 \tilde{u}_n}{\partial y^2} - c\frac{\partial^2 \tilde{u}_n}{\partial z^2} - D\right)\mathrm{d}\xi$$

$$\text{Case 3:}\quad u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \int_L^x \lambda_3 \left(\frac{\partial^2 \tilde{u}_n}{\partial t^2} - A\frac{\partial^2 u_n}{\partial \xi^2} - B\frac{\partial^2 \tilde{u}_n}{\partial y^2} - c\frac{\partial^2 \tilde{u}_n}{\partial z^2} - D\right)\mathrm{d}\xi.$$

Case 1: To make this correction functional stationary, notice that $\delta u_n(x, y, z, 0) = 0$,

$$\delta u_{n+1} = \delta u_n + \left(\lambda_1 (\delta u_n)'\right)_0^t - \left(\lambda_1' \delta u_n\right)_0^t + \int_0^t \lambda_1'' \delta u_n \mathrm{d}\xi = 0.$$

Its stationarity conditions can be obtained as follows:

$$\delta u_n : 1 - \lambda_1'(t) = 0,$$
$$\delta u_n' : \lambda_1(t) = 0,$$
$$\delta u_n : \lambda_1''(\xi) = 0.$$

From this the Lagrange multiplier can be identified as $\lambda_1 = \xi - t$, and the following iteration formula will be obtained:

$$u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \int_0^t (\xi - t)\left(\frac{\partial^2 u_n}{\partial \xi^2} - A\frac{\partial^2 u_n}{\partial x^2} - B\frac{\partial^2 u_n}{\partial y^2} - C\frac{\partial^2 u_n}{\partial z^2} - D\right)\mathrm{d}\xi. \tag{2}$$

Starting with u_0, which can be chosen arbitrarily, Eq. (2) will be used to derive iterative approximations to u. In this case, considering initial conditions, we use u_0 as $u_0(x, y, z, t) = f(x, y, z) + g(x, y, z)t$.

Case 2: To make this correction functional stationary, consider $\delta u_n(0, y, z, t) = 0$,

$$\delta u_{n+1} = \delta u_n - \left(\lambda_2 A\,(\delta u_n)'\right)_0^x + \left((\lambda_2 A)'\delta u_n\right)_0^x - \int_0^x (\lambda_2 A)''\delta u_n \mathrm{d}\xi = 0.$$

Its stationarity conditions can be obtained as follows:

$$\begin{aligned}&\delta u_n : 1 + (\lambda(x)A(x, y, z, t))' = 0,\\ &\delta u_n' : \lambda(x)A(x, y, z, t) = 0,\\ &\delta u_n : (\lambda(\xi)A(\xi, y, z, t))'' = 0.\end{aligned}$$

From this the Lagrange multiplier can be identified as $\lambda_2 = \frac{1}{A(\xi,y,z,t)}(x - \xi)$, and the following iteration formula will be obtained:

$$\begin{aligned}u_{n+1}(x, y, z, t) &= u_n(x, y, z, t)\\ &\quad + \int_0^x \frac{1}{A(\xi, y, z, t)}(x - \xi)\left(\frac{\partial^2 u_n}{\partial t^2} - A\frac{\partial^2 u_n}{\partial \xi^2} - B\frac{\partial^2 u_n}{\partial y^2} - C\frac{\partial^2 u_n}{\partial z^2} - D\right)\mathrm{d}\xi. \qquad (3)\end{aligned}$$

In this case, considering boundary conditions, we use u_0 as $u_0(x, y, z, t) = f_1(y, z, t) + g_1(y, z, t)x$.

In case 3, like for case 2, notice that $\delta u_n(L, y, z, t) = 0$; we obtain $\lambda_3 = \frac{1}{A(\xi,y,z,t)}(x - \xi)$, and the following iterative formula will be obtained:

$$\begin{aligned}u_{n+1}(x, y, z, t) &= u_n(x, y, z, t)\\ &\quad + \int_L^x \frac{1}{A(\xi, y, z, t)}(x - \xi)\left(\frac{\partial^2 u_n}{\partial t^2} - A\frac{\partial^2 u_n}{\partial \xi^2} - B\frac{\partial^2 u_n}{\partial y^2} - C\frac{\partial^2 u_n}{\partial z^2} - D\right)\mathrm{d}\xi. \qquad (4)\end{aligned}$$

Considering boundary conditions, we use u_0 as $u_0(x, y, z, t) = f_1(y, z, t) + g_1(y, z, t)(x - L)$.

Example 1. Consider the following equation with the following initial conditions:

$$\begin{cases}\dfrac{\partial^2 u}{\partial t^2} = A\dfrac{\partial^2 u}{\partial x^2} + B\dfrac{\partial^2 u}{\partial y^2} + C\dfrac{\partial^2 u}{\partial z^2} + D\\ u(x, y, z, 0) = a_1 x + a_2 x^2 + b_1 y + b_2 y^2 + c_1 z + c_2 z^2\\ \dfrac{\partial u}{\partial t}(x, y, z, 0) = a_1' x + a_2' x^2 + b_1' y + b_2' y^2 + c_1' z + c_2' z^2\end{cases}$$

where A, B, C and D are constant.

From (2), we have

$$u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \int_0^t (\xi - t)\left(\frac{\partial^2 u_n}{\partial \xi^2} - A\frac{\partial^2 u_n}{\partial x^2} - B\frac{\partial^2 u_n}{\partial y^2} - C\frac{\partial^2 u_n}{\partial z^2} - D\right)\mathrm{d}\xi. \qquad (5)$$

Starting with

$$u_0(x, y, z, t) = a_1 x + a_2 x^2 + b_1 y + b_2 y^2 + c_1 z + c_2 z^2 + (a_1' x + a_2' x^2 + b_1' y + b_2' y^2 + c_1' z + c_2' z^2)t$$

by iteration formula (5), we have

$$\begin{aligned}u_1(x, y, z, t) &= a_1 x + a_2 x^2 + b_1 y + b_2 y^2 + c_1 z + c_2 z^2 + (a_1' x + a_2' x^2 + b_1' y + b_2' y^2 + c_1' z + c_2' z^2)t\\ &\quad + \left(Aa_2 + Bb_2 + Cc_2 + \frac{1}{2}D\right)t^2 + (Aa_2' + Bb_2' + Cc_2')\frac{t^3}{3},\end{aligned}$$

which is an exact solution.

Example 2. Let us solve the following partial differential equations:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \dfrac{1}{6}\left(x^2\dfrac{\partial^2 u}{\partial x^2} + y^2\dfrac{\partial^2 u}{\partial y^2} + z^2\dfrac{\partial^2 u}{\partial z^2}\right) \\ u(x,y,z,0) = x^2y^2z^2 \\ \dfrac{\partial u}{\partial t}(x,y,z,0) = -x^2y^2z^2. \end{cases}$$

From (2) the variational iteration formula can be constructed as

$$u_{n+1}(x,y,z,t) = u_n(x,y,z,t) - \int_0^t (\xi - t)\left(\frac{\partial^2 u_n}{\partial \xi^2} - \frac{1}{6}\left(x^2\frac{\partial^2 u_n}{\partial x^2} + y^2\frac{\partial^2 u_n}{\partial y^2} + z^2\frac{\partial^2 u_n}{\partial z^2}\right)\right)\mathrm{d}\xi.$$

By imposing $u_0(x,y,z,t) = x^2y^2z^2 - x^2y^2z^2t$, we have the following approximate solutions:

$$u_1(x,y,z,t) = u_0 + x^2y^2z^2\left(\frac{t^2}{2!} - \frac{t^3}{3!}\right)$$

$$u_2(x,y,z,t) = u_0 + u_1 + x^2y^2z^2\left(\frac{t^4}{4!} - \frac{t^5}{5!}\right)$$

$$u_n(x,y,z,t) = u_0 + u_1 + \cdots + u_{n-1} + \left(\frac{t^{2n}}{(2n)!} - \frac{t^{2n+1}}{(2n+1)!}\right).$$

Thus we have $u(x,y,z,t) = \lim_{n\to\infty} u_n(x,y,z,t) = x^2y^2z^2\sum_{n=0}^{\infty}(-1)^n\frac{t^n}{n!} = x^2y^2z^2\mathrm{e}^{-t}$, which is an exact solution.

Example 3. Consider the following partial differential equation with specified initial conditions:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \mathrm{e}^{-xt}\dfrac{\partial^2 u}{\partial x^2} + \mathrm{e}^{-yt}\dfrac{\partial^2 u}{\partial y^2} + \mathrm{e}^{-zt}\dfrac{\partial^2 u}{\partial z^2} \\ u(x,y,z,0) = x^2 + y^2 + z^2 \\ \dfrac{\partial u}{\partial t}(x,y,z,0) = 0. \end{cases}$$

The variational iteration method consists of the following scheme:

$$u_{n+1}(x,y,z,t) = u_n(x,y,z,t) + \int_0^t (\xi - t)\left(\frac{\partial^2 u}{\partial^2 \xi} - \mathrm{e}^{-x\xi}\frac{\partial^2 u}{\partial x^2} + \mathrm{e}^{-y\xi}\frac{\partial^2 u}{\partial y^2} + \mathrm{e}^{-z\xi}\frac{\partial^2 u}{\partial z^2}\right)\mathrm{d}\xi.$$

Taking $u_0(x,y,z,t) = x^2 + y^2 + z^2$, we have

$$\begin{aligned} u_1(x,y,z,t) &= x^2 + y^2 + z^2 \\ &\quad + \frac{2\left(\left(\mathrm{e}^{-xt}(1+x) - 1\right)y^2z^2 + \left(\mathrm{e}^{-yt}(1+y) - 1\right)x^2z^2 + \left(\mathrm{e}^{-zt}(1+z) - 1\right)y^2x^2\right)}{x^2y^2z^2}. \end{aligned}$$

By using u_1 to compute u_2, and considering $u \approx u_2$, we derive

$$\begin{aligned} u(x,y,z,t) \approx{}& \frac{1}{y^6x^6z^6}(-2tz^6y^6x^5 - 24\mathrm{e}^{(-yt)}z^6x^6y + 2z^5y^6x^6 + 2\mathrm{e}^{(-2xt)}t^2y^6x^2z^6 + 24\mathrm{e}^{(-2zt)}y^6x^6z \\ &+ 24\mathrm{e}^{(-2xt)}y^6xz^6 + 24t\mathrm{e}^{(-zt)}y^6x^6z - 40\mathrm{e}^{(-zt)}y^6x^6 - 2tz^5y^6x^6 + 40\mathrm{e}^{(-2yt)}z^6x^6 \\ &+ 40\mathrm{e}^{(-2xt)}z^6y^6 + 12t\mathrm{e}^{(-2zt)}y^6x^6z^2 + 12t\mathrm{e}^{(-2xt)}y^6z^6x^2 + 16t\mathrm{e}^{(-2xt)}y^6z^6x \\ &+ 16t\mathrm{e}^{(-2zt)}y^6x^6z + 12t\mathrm{e}^{(-2yt)}y^2z^6x^6 + y^8z^6x^6 + z^8y^6x^6 + x^8z^6y^6 \\ &+ 16t\mathrm{e}^{(-2yt)}z^6x^6y + 24\mathrm{e}^{(-2yt)}z^6x^6y - 6ty^6z^6x^6 + 2y^5z^6x^6 + y^6z^6x^6 + 12t\mathrm{e}^{(-zt)}z^2x^6y^2 \\ &- 2ty^5z^6x^6 + 24t\mathrm{e}^{(-xt)}z^6y^6x + 40\mathrm{e}^{(-2zt)}x^6y^6 + 12t\mathrm{e}^{(-xt)}z^6y^6x^2 + 2\mathrm{e}^{(-2yt)}t^2z^6y^2x^6 \\ &+ 2\mathrm{e}^{(-2yt)}t^2z^6y^3x^6 - 40\mathrm{e}^{(-xt)}z^6y^6 + 2z^6y^6x^5 - 40\mathrm{e}^{(-yt)}z^6x^6 - 24\mathrm{e}^{(-xt)}z^6y^6x \\ &- 24\mathrm{e}^{(-zt)}x^6y^6z + 12t\mathrm{e}^{(-yt)}z^6y^2x^6 + 24t\mathrm{e}^{(-yt)}z^6yx^6 - 6t^2\mathrm{e}^{(-zt)}z^2y^6x^6 \end{aligned}$$

$$-6t^2e^{(-xt)}z^6y^6x^2 - 6t^2e^{(-yt)}z^3y^6x^6 + 6t^2e^{(-yt)}z^6y^6x^6 + t^2e^{(-zt)}z^6y^6x^6 + t^2e^{(-yt)}z^6y^6x^6 + 2t^2e^{(-2zt)}z^3y^6x^6 + 2t^2e^{(-2zt)}z^2y^6x^6 + 2t^2e^{(-2xt)}z^6y^6x^3).$$

It is worth pointing out that the results in Examples 1–3 are exactly the same as the results from applying the Adomian decomposition method [23].

Example 4. Consider the following equation with the following boundary conditions:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = A\dfrac{\partial^2 u}{\partial x^2} + B\dfrac{\partial^2 u}{\partial y^2} + C\dfrac{\partial^2 u}{\partial z^2} + D \\ u(0, y, z, t) = a_1 y + a_2 y^2 + b_1 z + b_2 z^2 \\ \dfrac{\partial u}{\partial x}(0, y, z, t) = a_1' y + a_2' y^2 + b_1' z + b_2' z^2 \end{cases}$$

where A, B, C and D are constant.

From (3), we have

$$u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \frac{1}{A}\int_0^x (x-\xi)\left(\frac{\partial^2 u_n}{\partial t^2} - A\frac{\partial^2 u_n}{\partial \xi^2} - B\frac{\partial^2 u_n}{\partial y^2} - C\frac{\partial^2 u_n}{\partial z^2} - D\right) d\xi. \tag{6}$$

Beginning with $u_0(x, y, z, t) = a_1 y + a_2 y^2 + b_1 z + b_2 z^2 + (a_1' y + a_2' y^2 + b_1' z + b_2' z^2)x$, by iteration formula (6), we have

$$u_1(x, y, z, t) = a_1 y + a_2 y^2 + b_1 z + b_2 z^2 + (a_1' y + a_2' y^2 + b_1' z + b_2' z^2)x - \frac{1}{A}\left(\left(a_2 B + b_2 C + \frac{1}{2}D\right)x^2 + \left(a_2' B + b_2' C\right)\frac{x^3}{3}\right),$$

which again is an exact solution.

Example 5. In this example we consider the following partial differential equations:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial y^2} + z\dfrac{\partial^2 u}{\partial z^2} \\ u(1, y, z, t) = yzt^2 \\ \dfrac{\partial u}{\partial x}(1, y, z, t) = -yzt^2. \end{cases}$$

From (4) the variational iteration formula will be obtained as

$$u_{n+1}(x, y, z, t) = u_n(x, y, z, t) + \int_1^x \frac{1}{\xi}(x-\xi)\left(\frac{\partial^2 u_n}{\partial t^2} - \xi\frac{\partial^2 u_n}{\partial \xi^2} + y\frac{\partial^2 u_n}{\partial y^2} + z\frac{\partial^2 u_n}{\partial z^2}\right) d\xi.$$

By imposing $u_0(x, y, z, t) = 2yzt^2 - xyzt^2$ we have

$$u_1(x, y, z, t) = 2yzt^2 + 4xyz\ln x - x^2yz - 2xyz + 3yz,$$

which is an exact solution.

3. Conclusion

In this work, we present the analytical approximation to a solution for wave equations in three different cases. We have achieved this goal by applying He's variational iteration method. Using the variational iteration method, it is possible to find the exact solution or a good approximate solution of the equation. It can be concluded that He's variational iteration method is very powerful and efficient technique for finding exact solutions for wide classes of problems. One of the restrictions in this method is the selection of u_0 relative to initial and boundary conditions. The suitable selection of u_0 has a very significant effect in computing u. Computations are performed using the Maple 10 package.

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